No: ________________________

IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2025

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

Date: 29 April 2025 (one hour thirty minutes)

This paper contains SIX questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate how the examiners intend to distribute the marks for this paper.

This is a CLOSED BOOK Examination.

1. a) (i) Sketch in the answer book the signal $x_0(t) = 3u(t-1) - u(t-4)$.

[2]

(ii) Sketch in the answer book the signal $x_1(t) = -(t-4) \times [u(t-1) - u(t-4)]$.

[4]

(iii) State the equation that describes the signal $y(t)$ shown in *Figure Q1*.

[6]

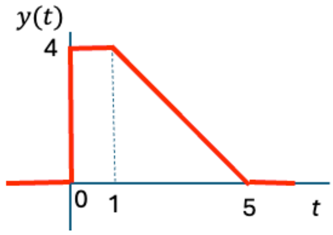


Figure Q1

b) A signal $x(t)$ is mathematically modelled by the following equation where $\delta(t)$ is the unit impulse function. Sketch in your answer book the signal $x(t)$.

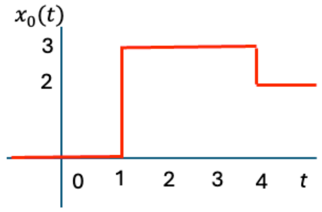
$$x(t) = 3\,\delta(t+1) - \delta(t) + 3\delta(t+2)$$

[3]

Solution to Q1

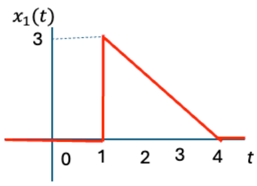
This question tests student's ability to model signal mathematically in terms of $u(t)$ and $\delta(t)$, and the application of the shift property of signals.

(a) (i) $x_0(t) = 3u(t-1) - u(t-4)$.



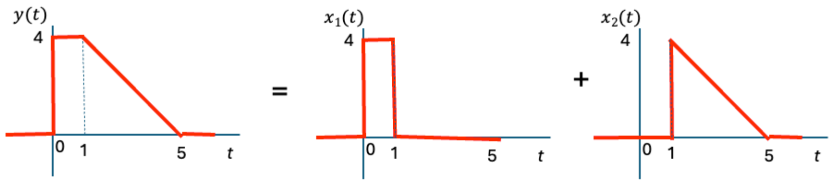
[2]

(ii) $x_1(t) = -(t-4) \times [u(t-1) - u(t-4)]$



[4]

(iii) $y(t)$ can be model by $y(t) = x_1(t) + x_2(t)$, as shown below.

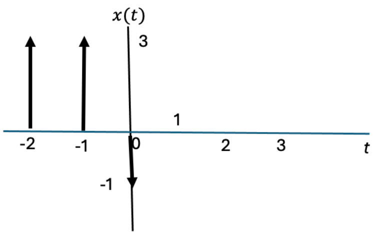


Hence, $y(t) = x_1(t) + x_2(t) = 4[u(t) - u(t-1)] - (t-5) \times [u(t-1) - u(t-5)]$

[6]

b)

$$x(t) = 3\,\delta(t+1) - \delta(t) + 3\delta(t+2)$$



[3]

Comments

(a) (i) Most students got this right.

(ii) This is trickier, but most students got this.

(iii) Many got this right, but some with minor errors.

(b) This is a "give away" because nearly everyone got this correctly.

2. A signal $y(t)$ is mathematically modelled by the following equation:

$$y(t) = \frac{1}{j}\left(e^{j(125.7t)} - e^{-j(125.7t)}\right) + 2.0$$

a) Rewrite $y(t)$ in terms of a sine function.

[3]

b) The signal is sampled at a sampling frequency of 160 Hz. It is known that the first sample y[0] = 2.0 and the second sample y[1] = 3.414. What are the values of $y[n]$ for $n = 2$, 3, 4 and 5?

[4]

c) Write in the answer book the mathematical equation that models the discrete signal $y[n]$.

[4]

d) A rectangular window is applied to the signal $y[n]$ to form a new signal $z[n]$ such that:

$$z[n] = \begin{cases} y[n] & \text{for } 0 \le n \le 5 \\ 0 & \text{otherwise} \end{cases}.$$

Write in the answer book the equation that describes the windowed signal $z[n]$ in terms of $\delta[n]$.

[4]

Solution to Q2

This question tests student's understanding of the relationship between discrete and continuous time signals, and the sinusoidal verses exponential form of signals.

(a)

$$y(t) = \frac{1}{j}\left(e^{j(125.7t)} - e^{-j(125.7t)}\right) + 2.0$$

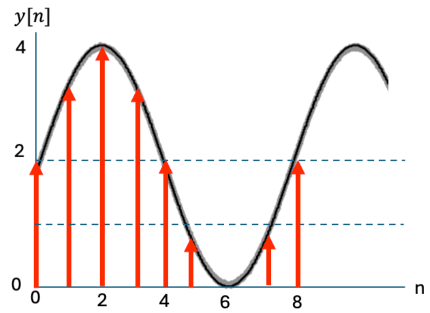
$$= 2\sin(125.7t) + 2.0$$

$$= 2\sin(2\pi \times 20t) + 2.0$$

[3]

(b) The sinewave frequency is 20Hz. Therefore, there are 8 samples per cycle. Each sample advances the phase by $\frac{2\pi}{8}$ or $\pi/4$. Given that y[0] = 2.0 and y[1] = 3.414, we know the phase angle of the sinewave is 0 when first sample was taken. Hence, subsequent sample for n=2 to 5 are:

y[2] = 2+2 = 4, y[3] = 2+1.414=3.414, y[4] = 2.0, y[5] = 2 – 1.414 = 0.586.



[4]

(c) With signal frequency at 20Hz and sampling frequency of 160Hz, each sampling interval will increase the angle by $\frac{\pi}{4}$. Hence the equation is:

$$y[n] = 2.0 \times \sin\left(\frac{\pi}{4}n\right) + 2.0$$

[4]

(d) Based on the answer in (b), the equation is:

$$w[n] = 2\delta[n] + 3.414\,\delta[n-1] + 4\delta[n-2] + 3.414\delta[n-3] + 2\delta[n-4] + 0.586\,\delta[n-5]$$

[4]

Comments

- **(a)** Nearly everyone got this right. It is a straightforward application of the Euler's formula.
- (b) Most students got this correctly. The key is to realize that there are 8 samples per cycle. This help with the remaining parts of the questions and is the key to think about discrete sinusoidal signals and discrete frequencies.
- (c) About half of the students go this. The idea is that everyone increment of n (sample) you increase the PHASE of the sine wave by pi/4 radians.
- (d) Most students got (b) also got this one correctly.

3. You are required to design a device that detects sound made by blue whales in some part of the North Atlantic Ocean. It is known that blue whales produce sound in the frequency range of 10 to 40 Hz. An A-to-D converter (ADC) is used to convert the sound signal for processing with a microprocessor such that it can pick up whale sounds with loudness between 30 and 100 decibels (dB).

a) Someone suggests that a sampling frequency of 200Hz should be used. Explain your opinion on this suggestion and justify your answer.

[4]

b) If the microphone signal is sampled at 200 samples per second and the sound of a blue whale is detected, sketch the one-sided amplitude spectrum (i.e. only positive frequencies) of the **sampled signal** between 0Hz and 250Hz. State any assumptions used.

[6]

c) If the microphone amplifier has an automatic gain control that always adjusts the signal voltage to a full voltage range corresponding to 100 decibels, estimate the resolution of the ADC required in terms of number of bits? State any assumptions used and justify your answer.

[5]

d) Someone switches on an underwater beacon which emits two tones at 120 Hz and 150 Hz respectively. If no anti-aliasing filter is used, what **aliased frequency** components will be produced?

[5]

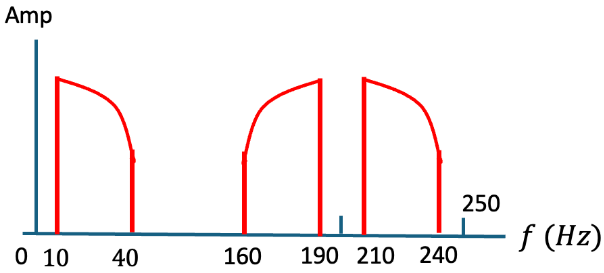
Solution to Q3

This question tests student's understanding the sampling theorem, how sampling changes the frequency signal, signal dynamic range and its relationship to ADC resolution and the consequence of aliasing.

a) Assuming that no signal is present except that of the blue whale, 200Hz sampling frequency is well above the minimum sampling rate of 80 samples per second. It is also not so high to result in unnecessary computation.

[4]

b) Assume an arbitrary spectral profile for the whale call signal as shown here between the frequency range of 10 – 40 Hz, and no other signal is present.



[6]

c) The automatic gain amplifier will always amplify the whale signal to a peak of 100dB. The assumption is that when the maximum signal is 100dB, the smallest signal that the ADC should detect is 30dB. That means the range of ADC is (100-30)=70 dB. That is, it should give a total range of 1 in 70dB.

The linear range for 70dB is $10^{\frac{70}{20}}$ = 3.162.3. Therefore, the ADC must be able to cope with a signal range of 1 in 3162.3. Assuming that this is the full signal range due to the automatic gain control, one would need 12 bits ADC, which provides a range of 1 in 4096.

[5]

d) The 120Hz signal is 20Hz beyond 100Hz, the maximum signal frequency that would not cause aliasing. Therefore, 120Hz is folded to a signal 20Hz **below** 100Hz, or 80Hz. Similarly, the 150Hz signal is folded to 50Hz.

[5]

Comments

a) Everyone got this right – it is almost a "give away" question.

b) Some students lost 1 mark because they only show a single frequency signal while this is a band. Some also lost 3 marks by only showing the upper band (210 to 240 Hz) or the lower band (160 to 190 Hz). But majority got this right.

c) This turns out to the part of the question that most students struggle. Mainly because the logic is not straight forward, and it is something new to most people. The question allow students to put forward their assumption and justify their answer. Many students just gave an answer without any reasoning behind it. Very few students scored marks for this question.

d) Most student who understood aliasing and how aliased components fold back got this correct.

4. A second-order LTI system has a transfer function H(s) of the general form:

$$H(s) = \frac{b_0}{s^2 + a_1 s + a_0} = K \frac{{\omega_0}^2}{s^2 + 2\zeta\omega_0 s + {\omega_0}^2}$$

where K = dc gain
ω_0 = natural frequency
ζ = damping ratio

Figure Q4 shows the small signal frequency response of a particular second-order system, with input $x(t)$ and output $y(t)$, measured at some input operating voltage x_0. It is also known that ζ = 0.1.

a) Assuming that this system is linear, derive the transfer function $H(s) = \frac{Y(s)}{X(s)}$ of the system based on information given.

[10]

b) Write down the differential equation that relates the output $y(t)$ to the input $x(t)$.

[4]

c) It was later discovered that the system input-output DC characteristic is nonlinear. The output y is a function of x according to the equation: $y = x^{2.5}$. What is the value of the input operating voltage x_0 around which the small signal frequency response was measured?

[6]

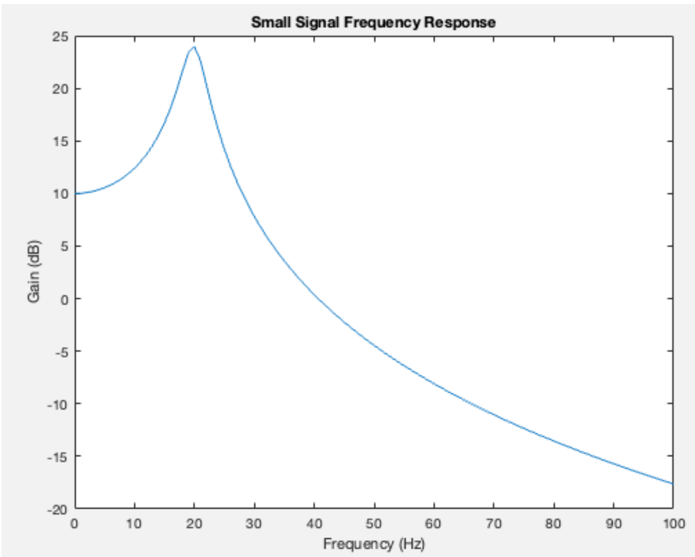


Figure Q4

Solution to Q4

This question tests student's understanding the characteristics of a 2nd order system, its transfer function, natural frequency, dc gain and the damping ratio. It also tests the understand about small signal frequency response and meaning of "gain" at the operating point of the system.

(a) The frequency response indicates two important characteristics of the system:

1) The gain of the system peaks at 20 Hz. Given that the damping ratio is small (0.1), this is approximately the natural frequency. Hence $\omega_0 = 2\,\pi f_0 = 125.7$ rad/s.
2) The DC gain is 10dB, i.e. K = $10^{0.5}$ = 3.16.

Therefore $a_0 = \omega_0^2 = 15.8\ \times 10^3$, $a_1 = 2 \times 0.1 \times 125.7 = 25.14$.

Further, $b_0 = K\omega_0^2 = 3.16 \times 15.8 \times 10^3 = 50 \times 10^3$.

Therefore, the transfer function is:

$$H(s) = \frac{50 \times 10^3}{s^2 + 25.14s + 15.8\ \times 10^3}$$

[10]

(b) The differential equation is:

$$\frac{d^2y(t)}{dt^2} + 25.14\frac{dy(t)}{dt} + 15.8\ \times 10^3\ y(t) = 50 \times 10^3\ x(t).$$

[4]

(c) The DC gain of the system is 10dB or 3.16. Given that $y = x^{2.5}$, and that the gain of the system at the operating point x_0 is given by the gradient of the output vs input characteristics at this input value, we need to find the value of x such that the gradient of the function $y = x^{2.5}$ is 3.16.

$\frac{dy}{dx} = 2.5x^{1.5} = 3.16.$ Therefore $x^{1.5} = \frac{3.16}{2.5} = 1.264.$

Solving this equation gives $x_0 \approx 1.17$

[6]

Comments

I was surprised how many students found this question difficult. Since this is derived partly from Lab 3, I was expecting this to be fairly easy, but I was wrong.

a) Almost everyone got wrong was believing that DC gain of 10dB is the same as a gain of 10. This is something that everyone should remember – gain of 2 is 6dB and gain of 10 is 20dB because in voltage gain dB = 20 $\log_{10}$ linear gain. Secondly, the majority students forgotten that ω_0 in the equation is in unit of rad/sec, and cycle/sec (Hz).

b) This is straight forward. Most students got this correct but using the wrong values from a). This is not penalized.

c) This is the hardest part of the questions and only a small number of students got this correctly. It is in fact the direct application of Lab 3 with a different exponent (2.5 instead of 2). Student using the gain of 10 would get the answer of 2.5, and they were not penalized.

5. A digital filter has discrete input signal $x[n]$ and output signal $y[n]$, and the system is causal. The filter has a difference equation given by:

$$y[n] = 0.6\,x[n] + 0.4\,y[n-1]$$

a) Given that $x[n]$ is a unit step signal and that $y[-1] = 0$, list the values of $x[n]$ and $y[n]$ for n = -1, 0, 1, .. , 6.

[5]

b) Explain, with justification, the type of filtering the system is performing.

[2]

(i) Derive the transfer function $H[z]$, of this system in the z-transform domain.

[4]

(ii) Draw a diagram showing how this filter can be implemented using multipliers, adders and delay modules.

[4]

Solution to Q5

This question tests student's understanding of discrete signals, step response of discrete system and how digital filter is implemented with basic computational elements.

(a)

n	x[n]	0.4*y[n-1]	y[n]
-1	0	0.000	0.000
0	1	0.000	0.600
1	1	0.240	0.840
2	1	0.336	0.936
3	1	0.374	0.974
4	1	0.390	0.990
5	1	0.396	0.996
6	1	0.398	0.998

[5]

(b) This is a low pass filter because the impulse response rises gradually towards the final output value of 1.0 in what appears to be exponential like manner.

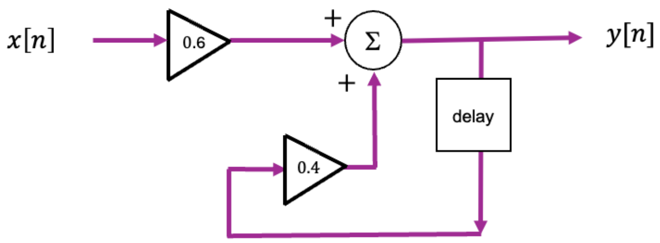
[2]

(c) The transfer function in z-domain is:

$$H(z) = \frac{0.6}{1 - 0.4z^{-1}}$$

[4]

(d)



$$y[n] = 0.6\,x[n] + 0.4\,y[n-1]$$

[4]

Comments

Most students found this question easy.

(a) Common error was using impulse function instead of unit step function as input.

(b) Many students answered high pass filter – which is completely wrong. Some answered IIR filter which earned 1 mark. IIR is a type of filter but does not describe the type of FILTERING it performs, which is asked by the question.

(c) Most students got this right.

(d) A few students took output y[n] AFTER the 0.4 multiplier, which is very wrong. Some also put the x0.6 modules after the adder, which is also totally wrong. Some subtract the feedback signal which is also totally wrong.

6. A DC motor system is controlled using pulse-width modulation with a duty cycle of $x(t)$, which has a range of 0 to 1. The speed of the motor $y(t)$ is measured in revolutions per second (rps), and the transfer function $G(s) = \frac{Y(s)}{X(s)}$ of the motor is given by:

$$G(s) = \frac{K_L}{0.05s + 1}$$

where K_L is a constant with a value of 4 if the system is ideal. However, manufacturing processes cause this value to vary by ±20%.

a) If the motor is controlled directly as an open-loop system with $x(t)$ set to 0.5 (i.e. 50% duty cycle), calculate the maximum and minimum steady-state speed of the motor.

[4]

b) Sketch the response of the open-loop system if x(t) is a step function $u(t)$. What is the time constant of the system?

[5]

c) *Figure Q6* shows a proportional closed-loop control system to control the motor described above. The proportional gain K_P is 10. Derive the closed-loop transfer function of this feedback system.

[6]

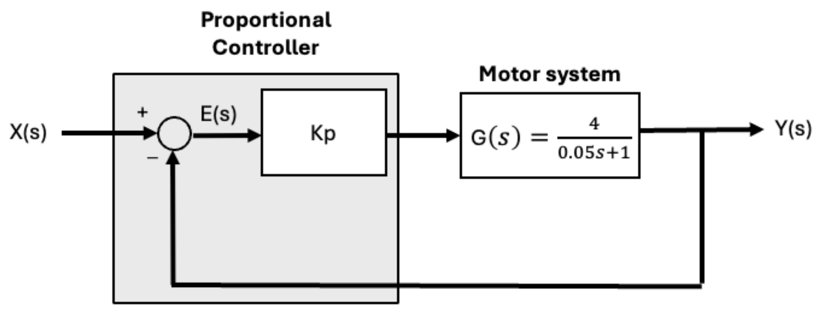


Figure Q6

Solution to Q6

This question tests students understanding of feedback control systems. It tests their understanding of proportional control and its ability to reduce the impact of the error in the gain of a system.

a) Nominal speed is 2, but due to variation of K_L, this may vary by ±0.4. Therefore, the maximum and minimum speed of motor would be 1.6 to 2.4 revolutions per second.

[4]

b) This requires students to know that: 1) the transfer function indicates a first order system; 2) its step response is an exponential rise with a time-constant equals to the coefficient value of the s term. Therefore, the time-constant is 50ms. Sketching is now very easy – the exponential signal reaches 63% of 4, or 2.52 at 0.05 second, and eventually reaches 4 in steady state.

[5]

c) The closed-loop transfer function is derived as follow:

$$E(s) = X(s) - Y(s) = Y(s)/K_pG(s)$$

$$Therefore, \quad Y(s)\left[\frac{1}{K_pG(s)} + 1\right] = X(s)$$

$$Y(s)\left(\frac{1 + K_pG(s)}{K_pG(s)}\right) = X(s)$$

Hence,

$$H_{CL}(s) = \frac{Y(s)}{X(s)} = \frac{Loop\_Gain}{1 + Loop\_gain} = \left(\frac{K_pG(s)}{1 + K_pG(s)}\right)$$

Therefore

$$H(s) = \frac{\frac{4K_p}{0.05s + 1}}{1 + \frac{4K_p}{0.05s + 1}} = \frac{40}{0.05s + 41} = \frac{40}{41} \times \frac{1}{\frac{0.05}{41}s + 1}$$

[6]

[END OF PAPER]

Comments

Most students got most of this right.

(a) Easy question. Some substitute 0.5 into s, the complex frequency, which is totally wrong. Some forgot that the duty cycle is 0.5, hence the speed was twice as should be and lose 2 marks.

(b) Many students lost 1 mark by not labelling the y-axis correctly (i.e. only show 63% at 0.05sec). Many did not show the exponential rise characteristic. Instead they had it as exponential fall after the step function.

(c) Most students got this perfectly.

Overall performance of the cohort on this paper:

	Q1	Q2	Q3	Q4	Q5	Q6	Exam	CW	**Module**
Average %	73	77	64	51	71	63	65.7	65.0	**65.4**
S.D. %	23	26	23	19	27	27	16.7	6.9	**11.4**
Max	15/15	15/15	20/20	19/20	15/15	15/15	93.0	78.0	**85.6**
Min	3/15	3/15	0	0	0	0	25	48.9	**40.0**

This examination paper achieved near perfect average mark (65.7%) and a wide standard deviation (16.8%). Combined with the course work component, the module average and spread are all pretty good. Further, the maximum examination marks of 93% shows that the paper is "do-able". At the same time, the minimum mark of 25% also show that it is not an easy paper unless one knows the stuff.

Every question has someone who scored full marks or zero marks (or near these extremes). This shows that no question is a "give away" but can be answered perfectly.